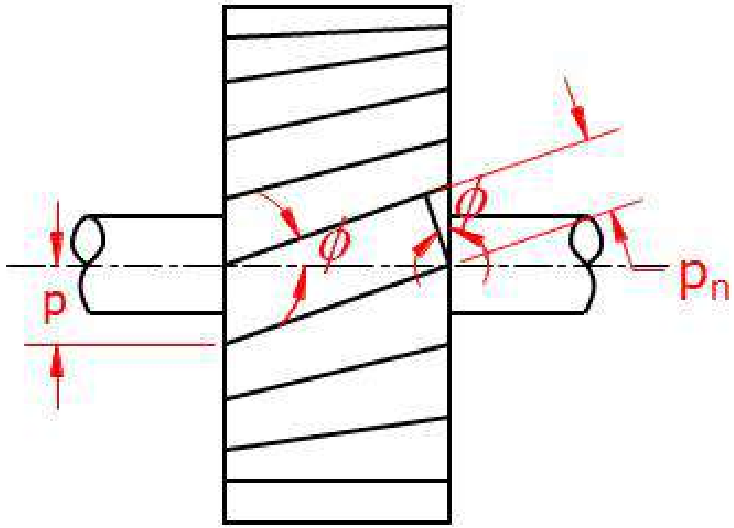
Helical Gears

2.1 Helical Gears : Basic difference between the spur gears and helical gears is teeth of spur gears are cut parallel to the axis, the teeth in helical gears are cut inclined to the axis in the form of a helix.



2.1.1 Helix angle : It is a constant angle made by the helices with the axis of rotation.

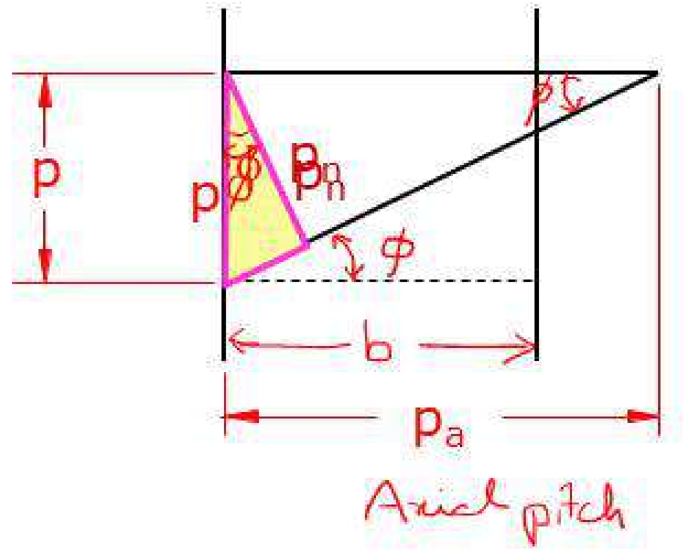
2.1.2 Transverse circular pitch (p) : It is the distance measured parallel to the axis, between similar faces of adjacent teeth.

2.1.3 Normal circular pitch (p_n) : It is the distance between similar faces of adjacent teeth measured normal to the teeth.

Axial pitch(p_a)

$$\tan\phi = \frac{p}{p_a}$$

$$p_a = \frac{p}{\tan\phi}$$



2.2 Normal Module (m_n)

$$Cos\phi = \frac{p_n}{p}$$

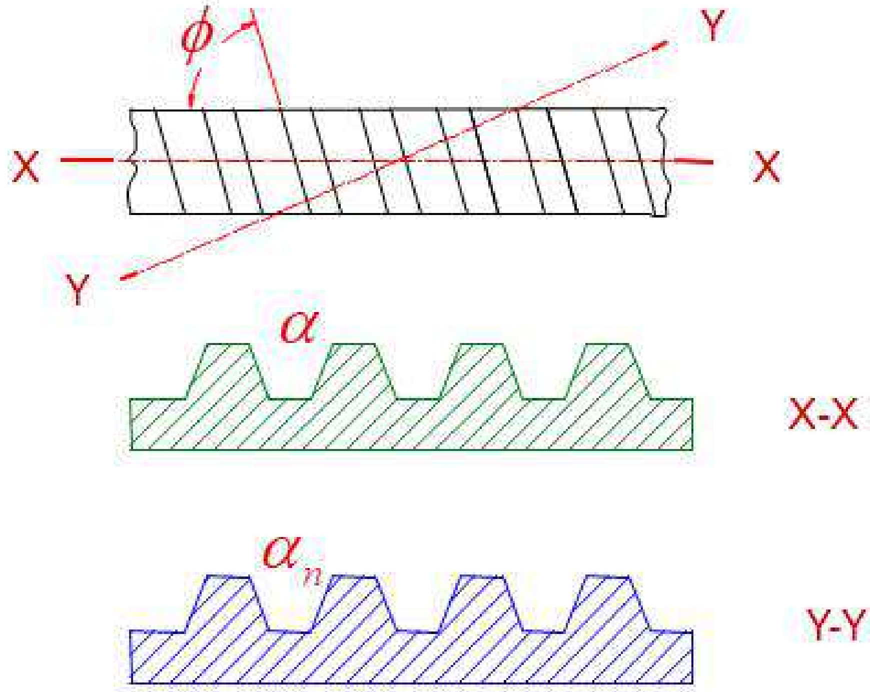
$$p_n = p.Cos\phi \quad -(A)$$

2.3 Pressure angles : There are two pressure angles

i) Transverse pressure angle α

ii) Normal pressure angle α_n

$$Cos\phi = \frac{\tan \alpha_n}{\tan \alpha}$$



2.4 Pitch Circle diameter(d)

$$d = m.Z$$

$$d = \frac{m_n}{Cos\phi}.Z \qquad \because m_n = m.Cos\phi$$

2.5 Centre to Centre distance

$$a = \frac{d_1}{2} + \frac{d_2}{2}$$

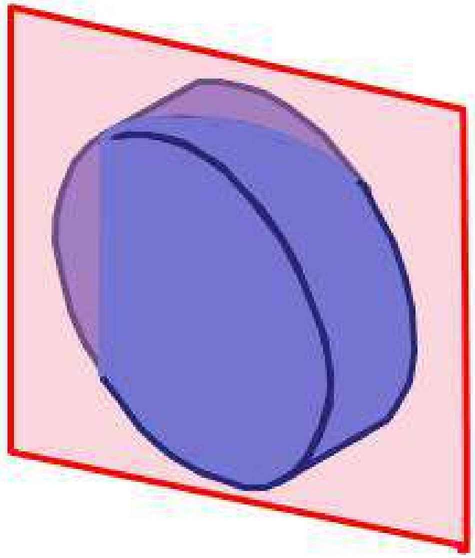
$$= \frac{Z_1.m_n}{2.Cos\phi} + \frac{Z_2.m_n}{2.Cos\phi}$$

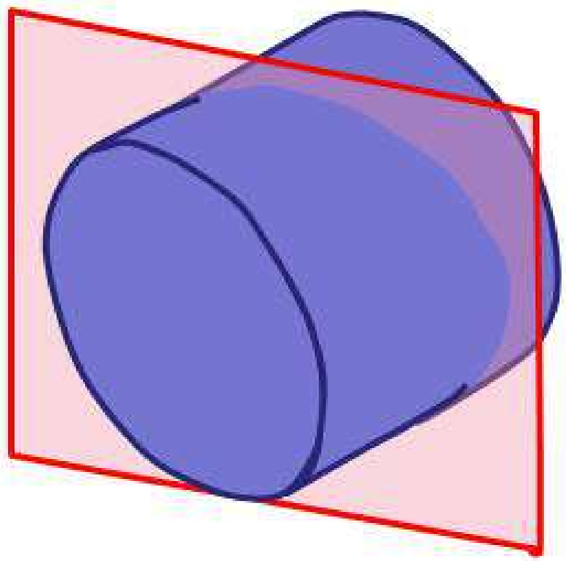
$$a = \frac{m_n(Z_1 + Z_2)}{2.Cos\phi}$$

2.6 Speed Ratio(i)

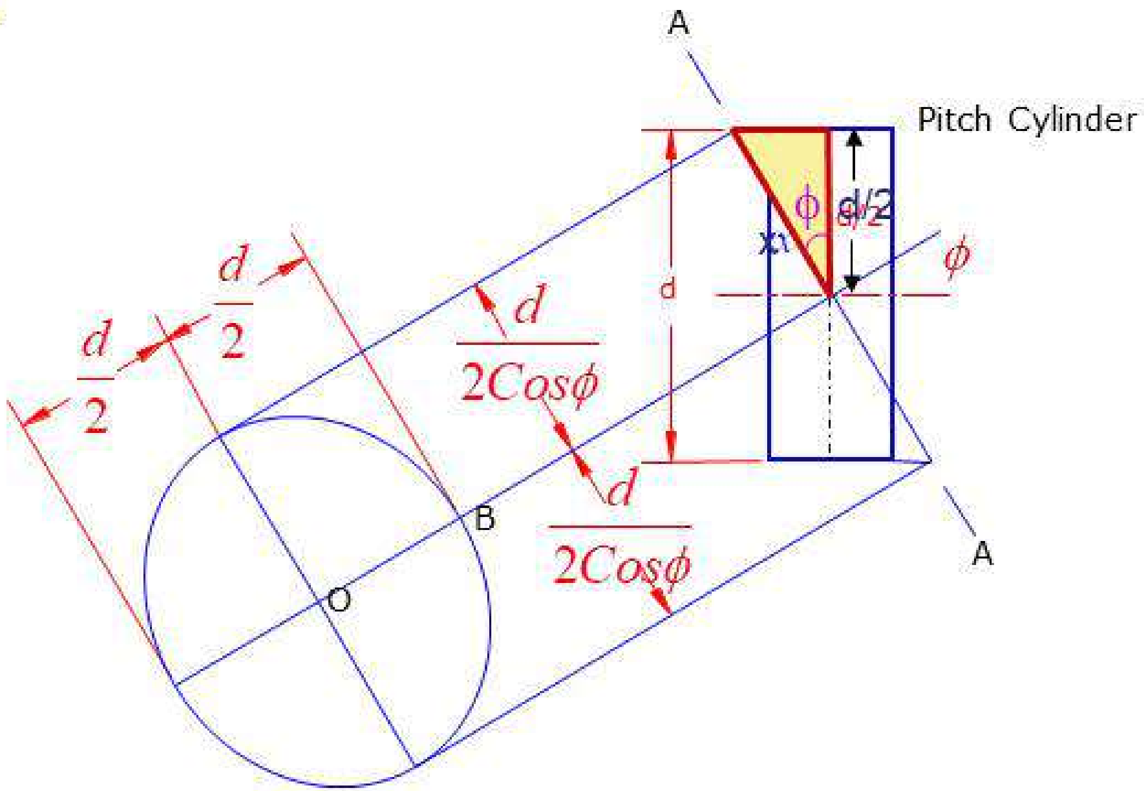
$$i = \frac{Z_g}{Z_p}$$

Virtual or Formative number of teeth : The virtual or formative no. of teeth of helical gear are the No. of teeth which can be generated on the surface of a cylinder having a pitch radius equal to the radius of curvature of a point at the tip of the minor axis of an ellipse, obtained by taking a section through the helical gear in a plane normal to the teeth.





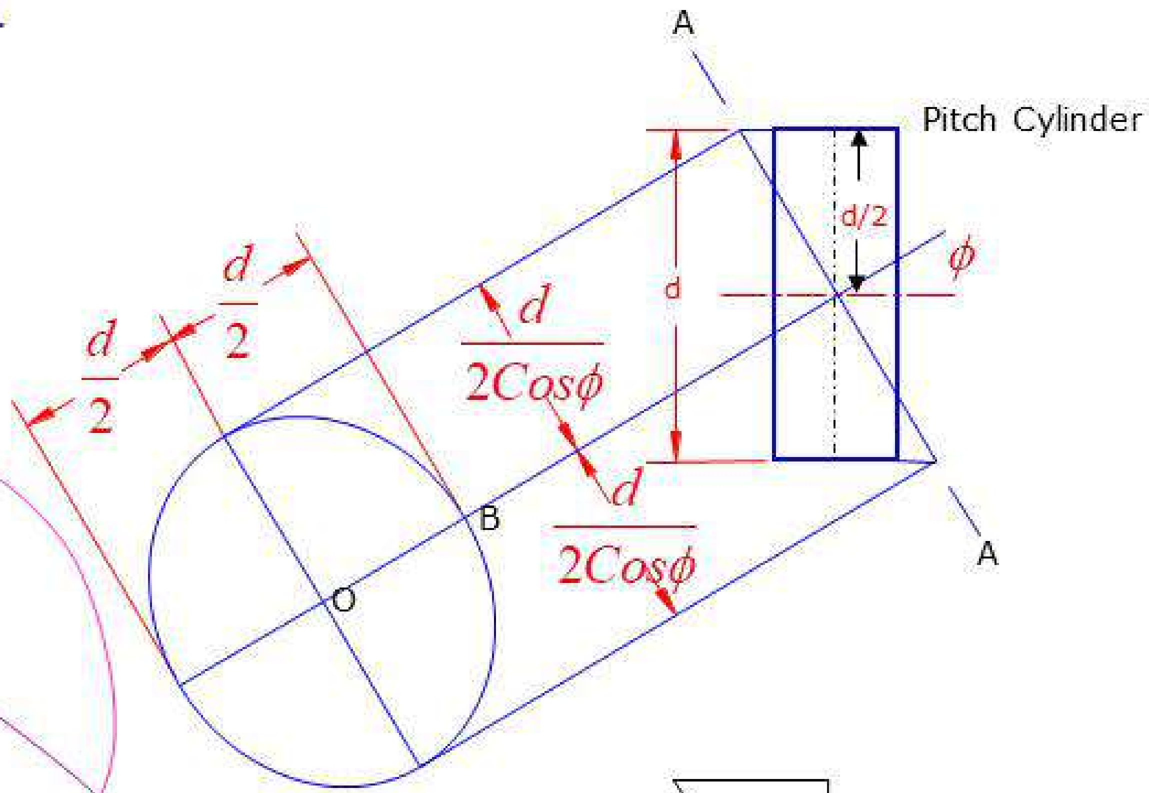
Virtual or Formative number of teeth : The virtual or formative no. of teeth of helical gear are the No. of teeth which can be generated on the surface of a cylinder having a pitch radius equal to the radius of curvature of a point at the tip of the minor axis of an ellipse, obtained by taking a section through the helical gear in a plane normal to the teeth.



$$Cos\phi = \frac{d}{2} \cdot \frac{1}{x}$$

$$x = \frac{d}{2Cos\phi}$$

Virtual or Formative number of teeth : The virtual or formative no. of teeth of helical gear are the No. of teeth which can be generated on the surface of a cylinder having a pitch radius equal to the radius of curvature of a point at the tip of the minor axis of an ellipse, obtained by taking a section through the helical gear in a plane normal to the teeth.



The intersection of the plane A-A and the pitch cylinder(Extended) produces an ellipse

$$Semi\ Major\ axis\,(a) = \frac{d}{2Cos\phi}$$

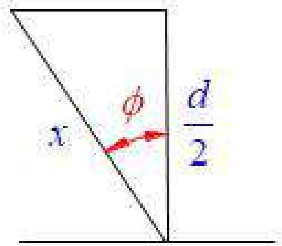
$$Semi\ Minor\ axis\,(b) = \frac{d}{2}$$

From the Analytical geometry radius of curvature r' at point B

$$r' = \frac{a^2}{b} = \frac{\dfrac{d^2}{4.Cos^2\phi}}{\dfrac{d}{2}}$$

$$Cos\phi = \frac{d}{2}.\frac{1}{x}$$

$$x = \frac{d}{2Cos\phi}$$



$$r' = \frac{a^2}{b}$$

$$= \frac{\frac{d^2}{4\cdot\cos^2\phi}}{\left(\frac{d}{2}\right)} = \frac{d}{2\cdot\cos^2\phi}$$

$$r' = \frac{d}{2\cdot\cos^2\phi}$$

Pitch Circle diameter of Formative Spur gear of module($\mathbf{m_n}$) is

$$d' = 2\cdot r' = 2\cdot\frac{d}{2\cdot\cos^2\phi} = \frac{d}{\cos^2\phi}$$

Number of teeth on Formative Spur gear of module(m_n) is

$$z' = \frac{d'}{m_n}$$

$$\because \quad d' = \frac{d}{\cos^2\phi}$$

$$= \frac{d}{\cos^2\phi} \cdot \frac{1}{m_n}$$

$$\because \quad d = \frac{m_n}{\cos\phi} \cdot z$$

$$= \frac{m_n}{\cos\phi} \cdot z \cdot \frac{1}{\cos^2\phi} \cdot \frac{1}{m_n}$$

$$= \frac{z}{\cos^3\phi}$$

2.8 Tooth Proportions : Standard Normal moduls

m_{n}(mm) 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8 and 10

$$Addendum\ (h_a) = m_n$$

$$Dedendum\,(h_f) = 1.25 m_n$$

$$Clearance\,(c) = 0.25 m_n$$

$$Addendum\ (d_a) = d + 2h_a = \frac{m_n}{Cos\phi}.Z + 2m_n$$

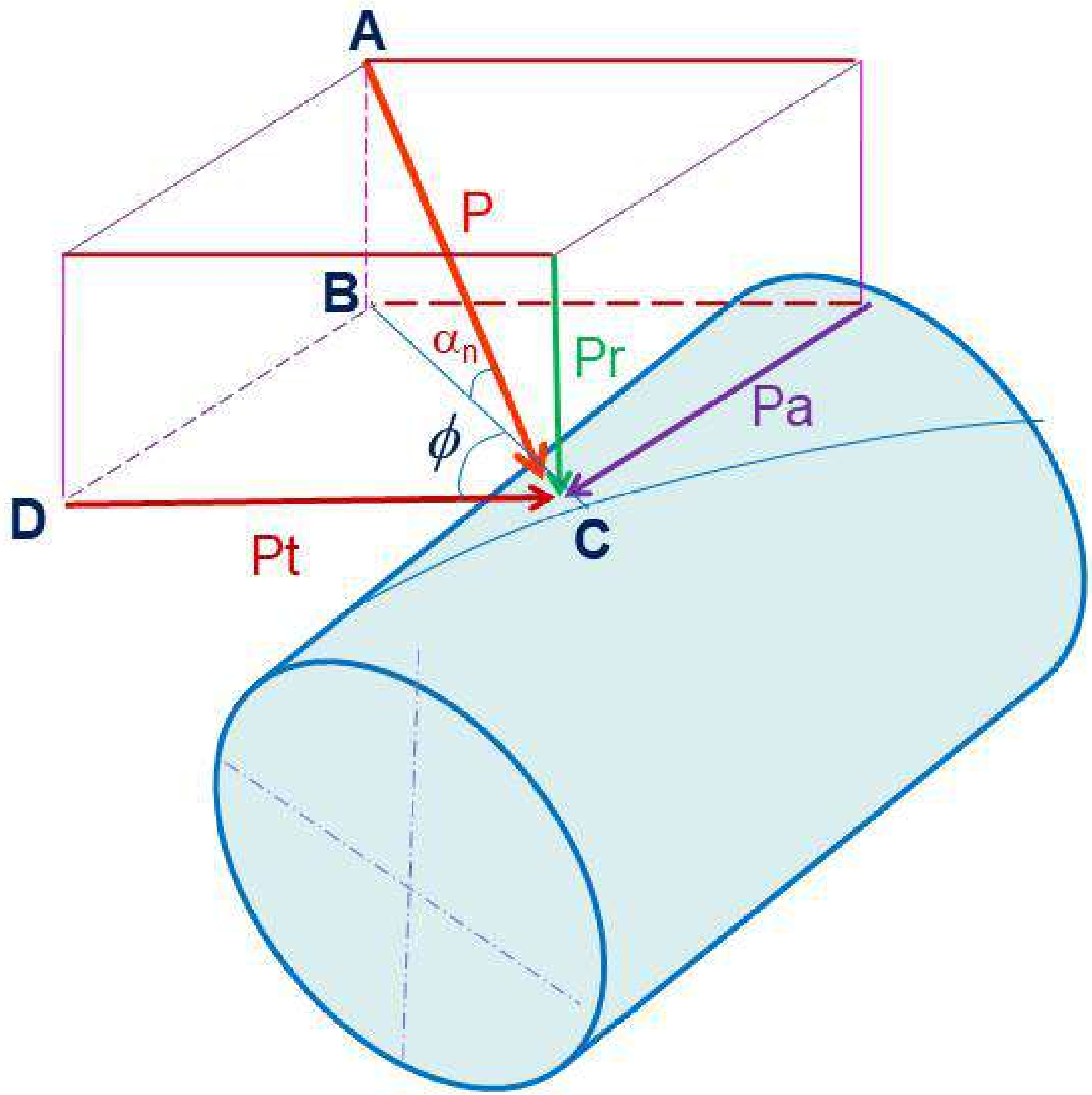
$$d_a = m_n \left[\frac{Z}{Cos\phi} + 2 \right]$$

$$Dedendum\,(d_f) = d - 2h_f = \frac{m_n}{Cos\phi}.Z - 2.5 m_n$$

$$d_f = m_n \left[\frac{Z}{Cos\phi} - 2.5 \right]$$

Force Analysis

The resultant force 'P' acting on the teeth of a helical gear is resolved into three components P_t, P_r and P_a as shown in Figure.



Where

Pt= Tangential component

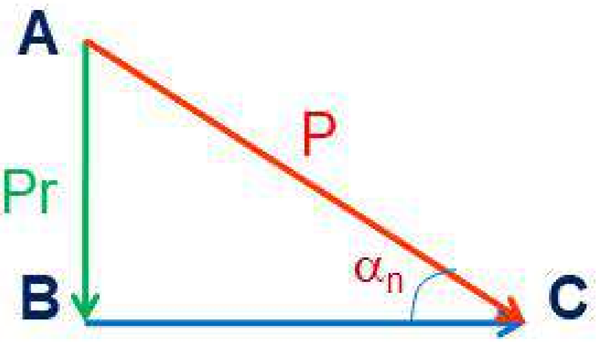
Pr= Radial Component

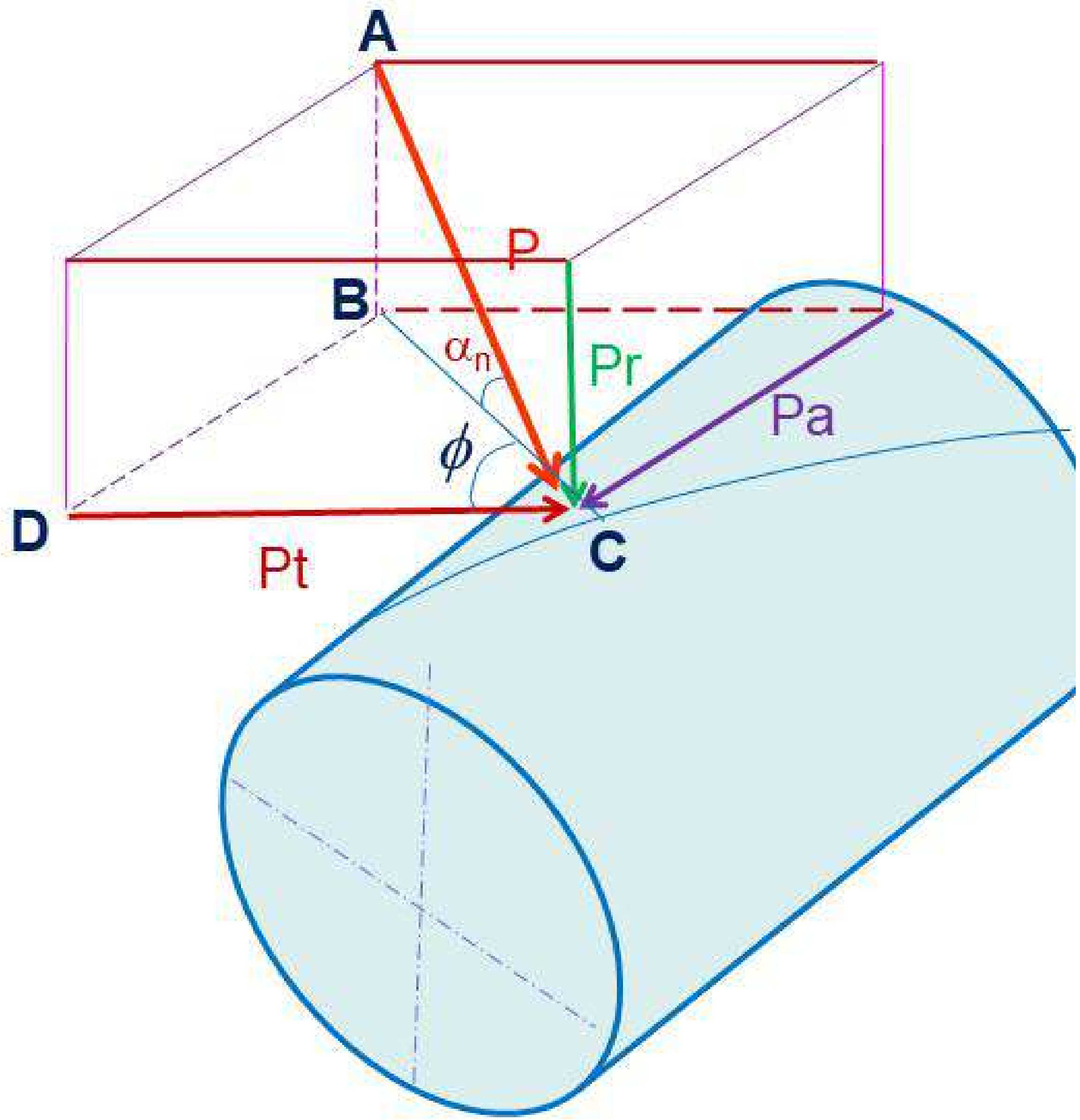
Pa= Axial or thrust component

α_n= Normal Pressure Angle , ϕ= Helix angle

Force Analysis

From $\triangle$ ABC



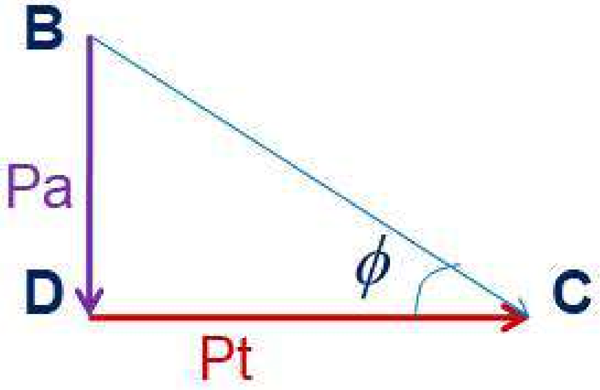


$P_r = P \, Sin\alpha_n$ ---- (1)

$BC = P \, Cos\alpha_n$ ---- (2)

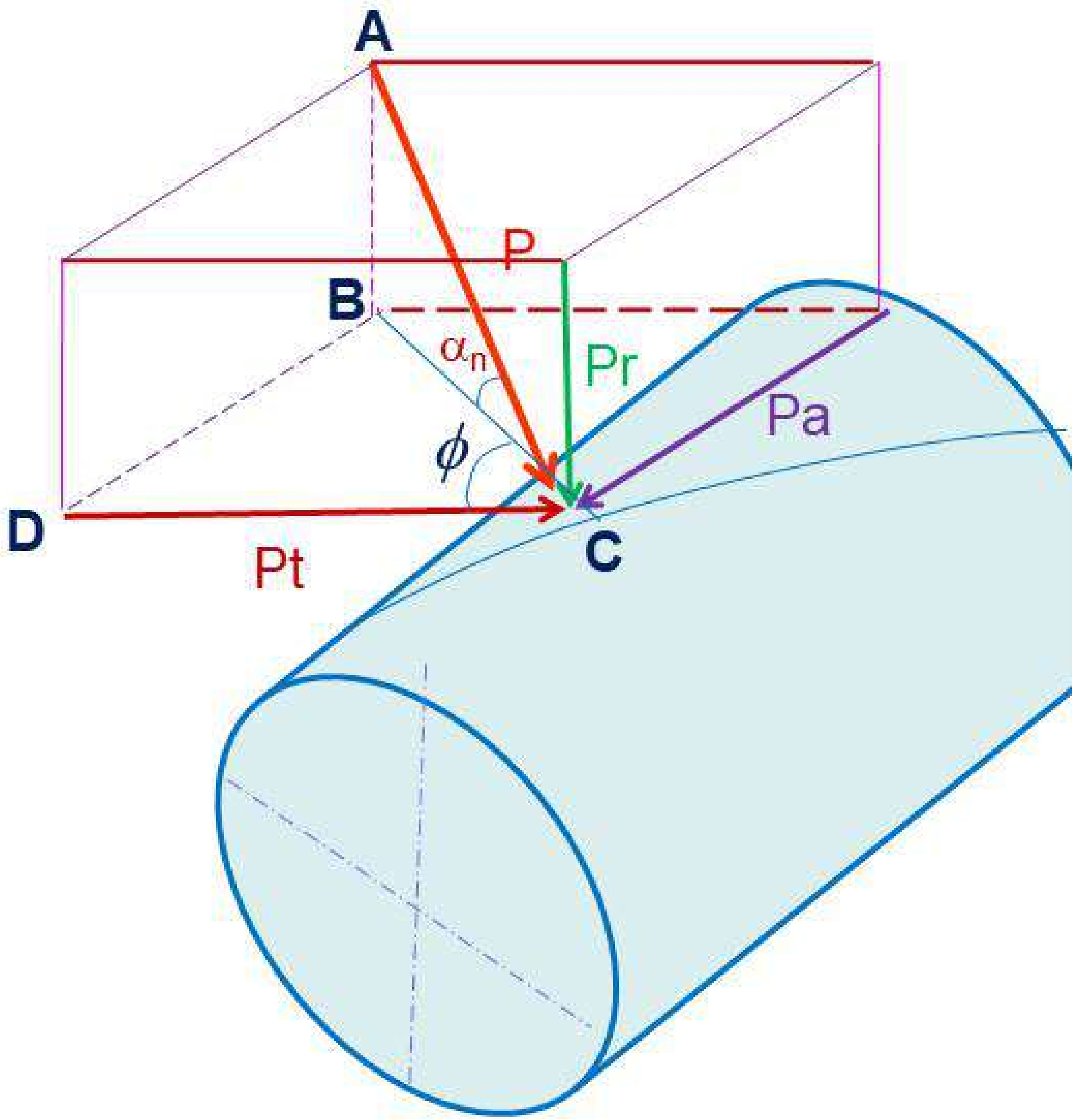
Force Analysis

From △ BDC



P_a=BC Sin ϕ ---- (3)

P_t=BC Cos ϕ ---- (4)



Substitute equation (2) in (3) and (4)

P_a=BC Sin ϕ ---- (3)

BC= P Cosα_n ---- (2)

P_t=BC Cos ϕ ---- (4)

P_a= P Cosα_n Sin ϕ ---- (a)

P_t= P Cosα_n Cos ϕ ---- (b)

Take (a)/(b)

$P_r = P \sin\alpha_n$ ---- (1)

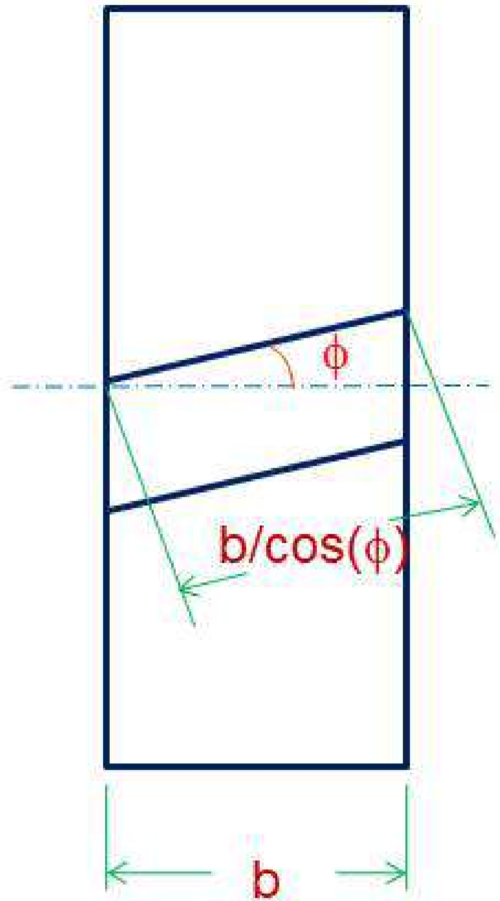
$P_t = P \cos\alpha_n \cos\phi$ ---- (b)   Take (1)/(b)

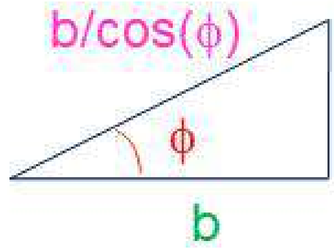
Beam Strength of Helical Gears

Consider Helical gear as equivalent formative spur gear.

Formative spur gear is an imaginary spur gear in a plane perpendicular to the tooth element..

Face width of Formative Spur gear





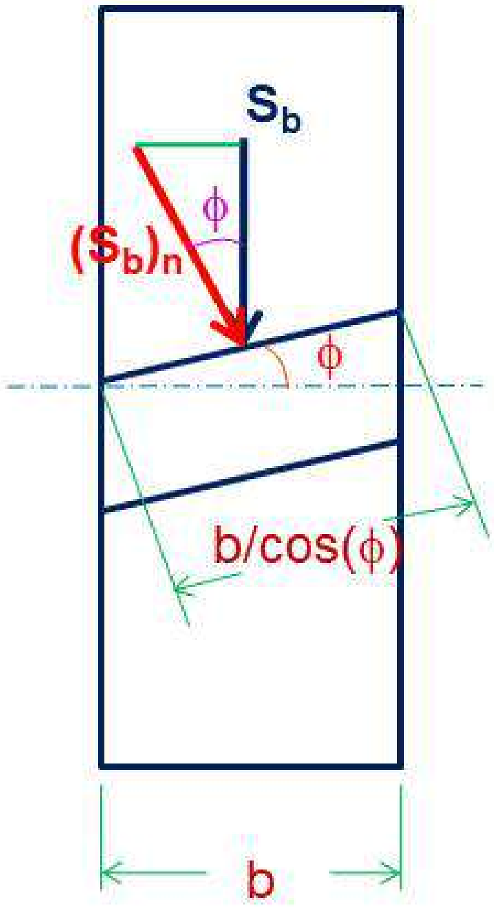
Beam Strength of Helical Gears

Beam Strength of Helical gear=S_b

Beam Strength of Formative spur gear=$(S_b)_n$..

$$\cos(\phi) = \frac{S_b}{(S_b)_n}$$

$$(S_b)_n = \frac{S_b}{\cos(\phi)}$$



For Formative spur Gear

Pitch circle diameter is d'

No.of teeth is Z'

Module is m_n

Beam Strength is $(S_b)_n$

Face width is $b/\cos\phi$

Y = Lewis form factor is based on virtual no. of teeth Z'

Now Substitute above values in $(S_b)_n = m_n . b' . \sigma_b . Y$

Effective load on gear tooth

Tangential component of force can be calculated from power, Then from Force analysis find Radial and Thrust components.

1. Effective load by **Barth Equation**

$$P_{eff} = \frac{C_s . P_t}{C_v}$$

$$ii)\ C_v = \frac{5.6}{5.6 + \sqrt{v}}$$

2. Effective load by **Spot's Equation**

$$P_{eff} = (C_s . P_t + P_d)$$

Where Pd = Dynamic load

Buckingham's equation for dynamic load

$$P_d = \frac{21v(Ceb.Cos^2\phi + P_t).Cos\phi}{21v + \sqrt{Ceb.Cos^2\phi + P_t}}$$

Where Pd = Dynamic load

C=Deformation factor

e= Sum of the errors of meshing teeth=ep+eg

$$C = \frac{K}{\left[\frac{1}{E_p} + \frac{1}{E_g}\right]}$$

Where K= constant that depends upon the form of tooth

Ep=Young's Modulus of pinion(N/mm^2)

Eg=Young's Modulus of gear(N/mm^2)

K=0.107 for 14.5^0 system

K=0.111 for 20^0 system

K=0.115 for 20^0 stub system

In order to avoid failure

$S_b > P_{eff}$

$S_b = fs. P_{eff}$

$$f_s = \frac{S_b}{P_{eff}}$$

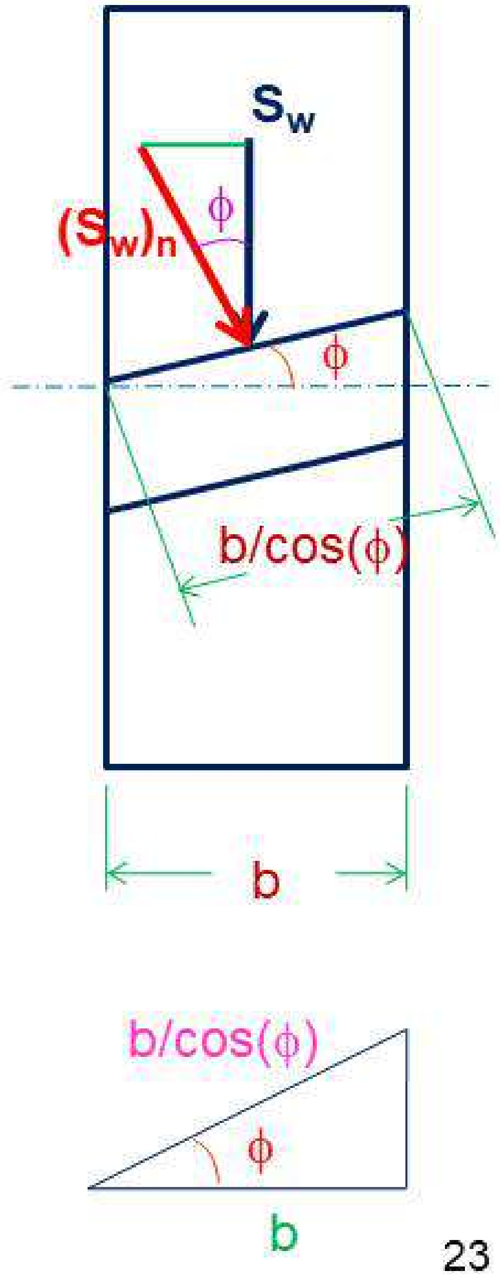
Wear Strength of Helical Gears

Consider Formative spur gear of Helical gear

$$(S_w)_n = b'.Q.d_p^1.K \qquad --(1)$$

$$\left.\begin{aligned} (S_w)_n &= \frac{S_w}{Cos\phi} \\ b' &= \frac{b}{Cos\phi} \\ d_p^1 &= \frac{d_p}{Cos^2\phi} \end{aligned}\right\} \quad (2)$$

Q and K remains same



Substitute (2) in (1)

$$\frac{S_w}{\cancel{Cos\phi}} = \frac{b}{\cancel{Cos\phi}}.Q.\frac{d_p}{Cos^2\phi}.K$$

$$S_w = \frac{b.Q.d_p.K}{Cos^2\phi}$$

Ratio Factor Q

$$Q = \frac{2.Z_g^1}{Z_g^1 + Z_p^1} \quad --(1)$$

$$Z_g^1 = \frac{Z_g}{Cos^3\phi} \quad --(2)$$

$$Z_p^1 = \frac{Z_p}{Cos^3\phi} \quad --(3)$$

Substitute (2) and (3) in (1)

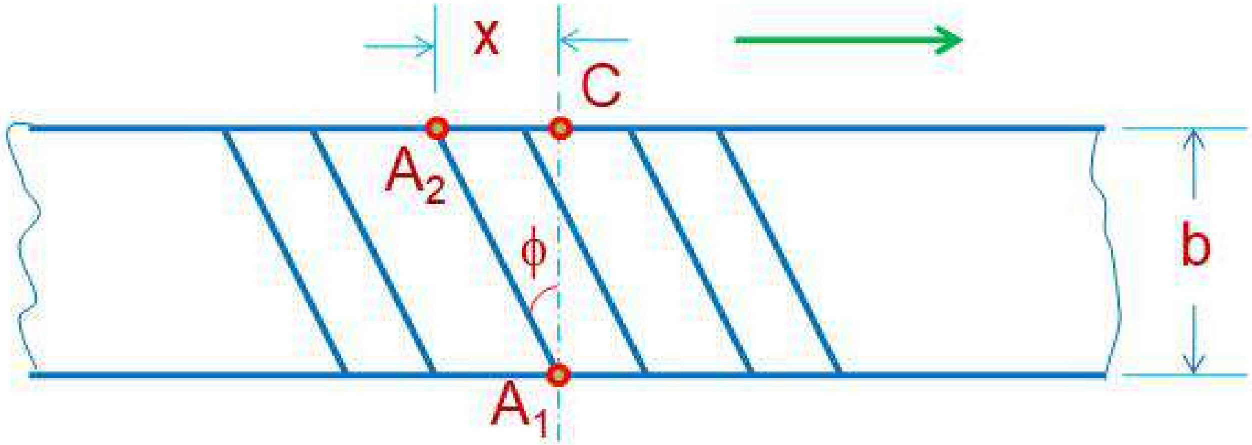
In order to avoid failure

$S_w > P_{eff}$

$S_w = fs.\ P_{eff}$

$$f_s = \frac{S_w}{P_{eff}}$$

Condition for minimum face width



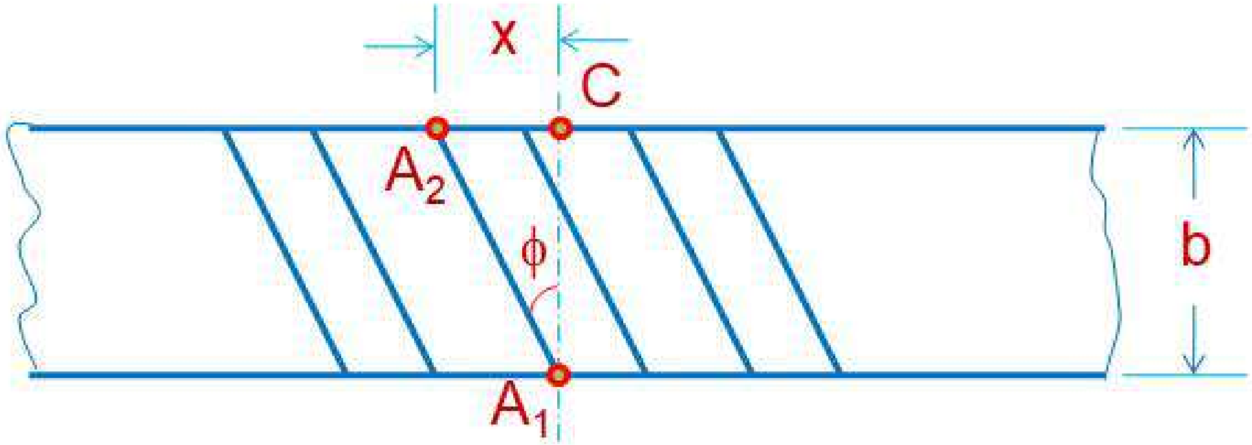
The first point come in contact with its meshing tooth on the other gear is called leading edge of the tooth. Here its is A1.

Last point come in contact with its meshing tooth on the other gear is called trailing edge of the tooth. Here its is A2.

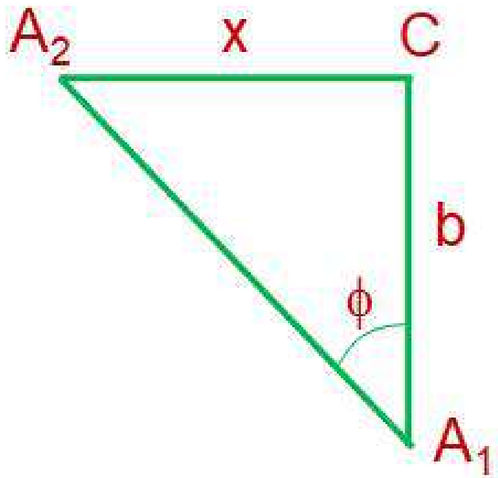
In order to have contact leading edge of the tooth should be advanced ahead of the trailing edge by a distance greater than circular pitch.

$$x \geq p \quad -(1)$$

Condition for minimum face width



To get x value consider Triangle A_1A_2C



$$\tan \phi = \frac{x}{b}$$

$$x = b.\tan \phi \quad -(2)$$

$$x \geq p \quad -(1)$$

$$p = \pi.m = \pi.\frac{m_n}{\cos\phi} \quad -(3)$$

$$x = b.\tan\phi \quad -(2)$$

Substitute (2) and (3) in (1)

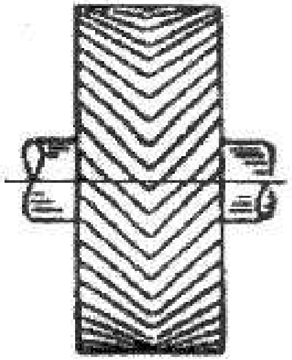
$$b.\tan\phi \geq \frac{\pi.m_n}{\cos\phi}$$

$$b \geq \frac{\pi.m_n}{\tan\phi.\cos\phi}$$

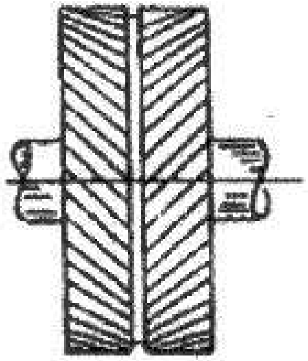
$$b \geq \frac{\pi.m_n}{\frac{\sin\phi}{\cancel{\cos\phi}}.\cancel{\cos\phi}}$$

$$b \geq \frac{\pi.m_n}{\sin\phi}$$

Herringbone Gear



Herringbone Gear



Double-Crossed
Helical Gear

The thrust forces can be eliminated by using Herringbone or Double helical gears.

Both Herringbone Gear and Double Helical gear consists of same module, No. of teeth and pitch circle diameter. But teeth having opposite hand of helix.

Helix angle for Normal gear is 12^0 to 25^0

Helix angle for Herringbone gear is 20^0 to 45^0

In design Herrigbone gear. Gear is considered as two identical gears each transmitting half of the power.